Some Bianchi Type-V Models of Accelerating Universe with Dark Energy

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Abstract

The paper deals with a spatially homogeneous and anisotropic universe filled with perfect fluid and dark energy components. The two sources are assumed to interact minimally together with a special law of variation for the average Hubble's parameter in order to solve the Einstein's field equations. The law yields two explicit forms of the scale factor governing the Bianchi-V space-time and constant values of deceleration parameter. The role of dark energy with variable equation of state parameter has been studied in detail in the evolution of Bianchi-V universe. It has been found that dark energy dominates the Universe at the present epoch, which is consistent with the observations. The Universe achieves flatness after the dominance of dark energy. The physical behavior of the Universe has been discussed in detail.

Keywords: Bianchi-V space-time, Hubble's parameter, Deceleration parameter, Dark energy.

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1 Introduction

Recent observations like type Ia supernova (SN Ia) [1]−[5], CMB anisotropy [6, 7], and large scale structure [8] strongly indicate that our Universe is spatially flat and there exists an exotic cosmic fluid called dark energy (DE) with negative pressure, which constitutes about 70 percent of the total energy of Universe. It is an irony of the nature and is a puzzling phenomenon that most abundant form of matter-energy in the Universe is most mysterious. Many cosmologists believe that the simplest candidate for the DE is the cosmological constant (Λ) or vacuum energy since it fits the observational data well. During the cosmological evolution, the Λ-term has the constant energy density and pressure $p^{(de)} = -\rho^{(de)}$. However, one has the reason to dislike the cosmological constant since it always suffers from the theoretical problems such as the "fine-tuning" and "cosmic coincidence" puzzles [9]. That is why, the different forms of dynamically changing DE with an effective equation of state (EoS), $\omega^{(de)} = p^{(de)}/\rho^{(de)} < -1/3$, have been proposed in the literature. Other possible forms of DE include quintessence ($\omega^{(de)} > -1$)[10], phantom ($\omega^{(de)} < -1$) [11] etc. While the possibility $\omega^{(de)} << -1$ is ruled out by current cosmological data from SNe Ia (Supernovae Legacy Survey, Gold sample of Hubble Space Telescope) [5, 12], CMBR (WMAP, BOOMERANG) [13, 14] and large scale structure (Sloan Digital Sky Survey) [15] data, the dynamically evolving DE crossing the phantom divide line (PDL) ($\omega^{(de)} = -1$) is mildly favored. Some other limits obtained from the observational results coming from SN Ia data (Knop et al [16]) collaborated with CMBR anisotropy and galaxy clustering statistics (Tegmark et al [17]) are $-1.67 < \omega^{(de)} < -0.62$ and $-1.33 < \omega^{(de)} < -0.79$ respectively.

Following Berman [18] and Kumar and Singh [19], recently Singh et al. [20] proposed a special law of variation for the average Hubble's parameter in Bianchi-V space-time, which yields a constant value of deceleration parameter. Such a law of variation for Hubble's parameter is not inconsistent with the observations and is also approximately valid for slowly time-varying DP models. The law provides explicit forms of scale factors governing the Bianchi-V Universe and facilitates to describe accelerating as well as decelerating modes of evolution of the Universe. Models with constant DP have been extensively studied in the literature in different contexts (see, Kumar and Singh [19] and references therein). Most of the models with constant DP have been studied by considering perfect fluid or ordinary matter in the Universe. But the ordinary matter is not enough to describe the dynamics of an accelerating Universe. This motivates the researchers to consider the models of the Universe filled with some exotic matter such as the DE along with the usual perfect fluid. Akarsu and Kilinc [21]−[23]

have investigated Bianchi-I and Bianchi-III DE models with constant DP. Yadav and Yadav [24] have studied the role of DE with variable EoS in Bianchi type-III Universe evolving with constant DP. Kumar [25, 26] has studied some isotropic and anisotropic models of accelerating Universe with DE and constant DP. Recently, Yadav et al [27] has presented LRS Bianchi-V Universe with DE characterized by variable EoS assuming constant DP.

In this paper, we have considered minimally interacting perfect fluid and DE energy components with constant DP within the framework of a Bianchi-V space-time in general relativity. The paper is organized as follows. In Section 2, the models and field equations have been presented. The Section 3 deals with the exact solutions of the field equations and physical behavior of the models. Finally, the results are discussed in section 4.

2 Model and field equations

The spatially homogeneous and anisotropic Bianchi-V space-time is described by the line element

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2\alpha x}(B^2 dy^2 + C^2 dz^2), \tag{1}$$

where A, B and C are the metric functions of cosmic time t and α is a constant.

We define $a = (ABC)^{\frac{1}{3}}$ as the average scale factor of the space-time (1) so that the average Hubble's parameter reads as

$$H = \frac{\dot{a}}{a}, \tag{2}$$

where an over dot denotes derivative with respect to the cosmic time t.

The Einstein's field equations in case of a mixture of perfect fluid and DE components, in the units $8\pi G = c = 1$, read as

$$R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij}, \tag{3}$$

where $T_{ij} = T_{ij}^{(m)} + T_{ij}^{(de)}$ is the overall energy momentum tensor with $T_{ij}^{(m)}$ and $T_{ij}^{(de)}$ as the energy momentum tensors of ordinary matter and DE, respectively. These are given by

$$T_{j}^{(m)\ i} = diag[-\rho^{(m)}, p^{(m)}, p^{(m)}, p^{(m)}] = diag[-1, \omega^{(m)}, \omega^{(m)}, \omega^{(m)}]\rho^{(m)} \tag{4}$$

and

$$T_{j}^{(de)\ i} = diag[-\rho^{(de)}, p^{(de)}, p^{(de)}, p^{(de)}] = diag[-1, \omega^{(de)}, \omega^{(de)}, \omega^{(de)}]\rho^{(de)} \tag{5}$$

where $\rho^{(m)}$ and $p^{(m)}$ are, respectively the energy density and pressure of the perfect fluid component or ordinary baryonic matter while $\omega^{(m)} = p^{(m)}/\rho^{(m)}$ is its EoS parameter. Similarly, $\rho^{(de)}$ and $p^{(de)}$ are, respectively the energy density and pressure of the DE component while $\omega^{(de)} = p^{(de)}/\rho^{(de)}$ is the corresponding EoS parameter.

In a comoving coordinate system, the field equations (3), for the Bianchi-I space-time (1), in case of (4) and (5), read as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = -\omega^{(m)}\rho^{(m)} - \omega^{(de)}\rho^{(de)}, \tag{6}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{\alpha^2}{A^2} = -\omega^{(m)}\rho^{(m)} - \omega^{(de)}\rho^{(de)}, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = -\omega^{(m)}\rho^{(m)} - \omega^{(de)}\rho^{(de)}, \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3\alpha^2}{A^2} = \rho^{(m)} + \rho^{(de)}. \tag{9}$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{10}$$

The energy conservation equation $T^{(de)\ ij}_{;j} = 0$ yields

$$\dot{\rho}^{(m)} + 3(1 + \omega^{(m)})\rho^{(m)}H + \dot{\rho}^{(de)} + 3(1 + \omega^{(de)})\rho^{(de)}H = 0. \tag{11}$$

3   Solution of Field Equations

Integrating (10) and absorbing the constant of integration in B or C, without loss of generality, we obtain

$$A^2 = BC. \tag{12}$$

Subtracting (6) from (7), (6) from (8), (7) from (8) and taking second integral of each, we get the following three relations respectively:

$$\frac{A}{B} = d_1 \exp\left(x_1 \int a^{-3} dt\right), \tag{13}$$

$$\frac{A}{C} = d_2 \exp\left(x_2 \int a^{-3} dt\right), \tag{14}$$

$$\frac{B}{C} = d_3 \exp\left(x_3 \int a^{-3} dt\right), \tag{15}$$

where d_1 , x_1 , d_2 , x_2 , d_3 and x_3 are constants of integration.

From equations (13)-(15) and (12), the metric functions can be explicitly written as

$$A(t) = a, \tag{16}$$

$$B(t) = ma \exp\left(l \int a^{-3} dt\right), \tag{17}$$

$$C(t) = m^{-1} a \exp\left(-l \int a^{-3} dt\right). \tag{18}$$

where

$$m = \sqrt[3]{d_2 d_3}, \quad l = \frac{(x_2 + x_3)}{3}$$

with

$$d_2 = d_1^{-1}, \quad x_2 = -x_1. \tag{19}$$

In order to solve the field equations completely, first we assume that the perfect fluid and DE components interact minimally. Therefore, the energy momentum tensors of the two sources may be conserved separately.

The energy conservation equation $T^{(m)\ ij}_{;j} = 0$, of the perfect fluid leads to

$$\dot{\rho}^{(m)} + 3(1 + \omega^{(m)})\rho^{(m)} H = 0, \tag{20}$$

whereas the energy conservation equation $T^{(de)\ ij}_{;j} = 0$, of the DE component yields

$$\dot{\rho}^{(de)} + 3(1 + \omega^{(de)})\rho^{(de)} H = 0. \tag{21}$$

Following Akarsu and Kilinc [21], we assume that the EoS parameter of the perfect fluid to be a constant, that is,

$$\omega^{(m)} = \frac{p^{(m)}}{\rho^{(m)}} = const., \tag{22}$$

while $\omega^{(de)}$ has been allowed to be a function of time since the current cosmological data from SNIa, CMB and large scale structures mildly favor dynamically evolving DE crossing the PDL as discussed in Section 1.

Now integration of equation (20) leads to

$$\rho^{(m)} = c_0 a^{-3(1 + \omega^{(m)})}, \tag{23}$$

where c_0 is a positive constant of integration.

Finally, we constrain the system of equations with a law of variation for the average Hubble parameter in Bianchi-V space-time proposed by Singh et al. [20], which yields a constant value of DP. The law reads as

$$H = Da^{-n}, \tag{24}$$

where $D > 0$ and $n \geq 0$ are constants. In the following subsections, we discuss the DE cosmology for $n \neq 0$ and $n = 0$ by using the law (24).

3.1 DE Cosmology for $n \neq 0$

In this case, integration of (24) leads to

$$a(t) = (nDt + c_1)^{\frac{1}{n}}, \tag{25}$$

where c_1 is a constant of integration.

Inserting (25) into (16)-(18), we get

$$A(t) = (nDt + c_1)^{\frac{1}{n}}, \tag{26}$$

$$B(t) = m(nDt + c_1)^{\frac{1}{n}} \exp\left[\frac{l}{D(n-3)}(nDt + c_1)^{\frac{n-3}{n}}\right], \tag{27}$$

$$C(t) = m^{-1}(nDt + c_1)^{\frac{1}{n}} \exp\left[\frac{-l}{D(n-3)}(nDt + c_1)^{\frac{n-3}{n}}\right], \tag{28}$$

provided $n \neq 3$.

Therefore, the model (1) becomes

$$ds^2 = -dt^2 + T^{\frac{2}{n}}\left(dx^2 + m^2 e^{2\alpha x + kT^{\frac{n-3}{n}}}dy^2 + m^{-2}e^{2\alpha x - kT^{\frac{n-3}{n}}}dz^2\right), \tag{29}$$

where $T = nDt + C_1$ and $k = \frac{2l}{D(n-3)}$.

The average Hubble's parameter (H), energy density $(\rho^{(m)})$ of perfect fluid, DE density $(\rho^{(de)})$ and EoS parameter $(\omega^{(de)})$ of DE, for the model (29) are found to be

$$H = D(nDt + c_1)^{-1}, \tag{30}$$

$$\rho^{(m)} = c_0(nDt + c_1)^{-\frac{3(1+\omega^{(m)})}{n}}, \tag{31}$$

$$\rho^{(de)} = 3D^2(nDt + c_1)^{-2} - l^2(nDt + c_1)^{-\frac{6}{n}} - 3\alpha^2(nDt + c_1)^{-\frac{2}{n}} - c_0(nDt + c_1)^{-\frac{3(1+\omega^{(m)})}{n}}, \tag{32}$$

$$\omega^{(de)} = \frac{(2n-3)D^2(nDt + c_1)^{-2} - l^2(nDt + c_1)^{-\frac{6}{n}} + \alpha^2(nDt + c_1)^{-\frac{2}{n}} - c_0\omega^{(m)}(nDt + c_1)^{-\frac{3(1+\omega^{(m)})}{n}}}{3D^2(nDt + c_1)^{-2} - l^2(nDt + c_1)^{-\frac{6}{n}} - 3\alpha^2(nDt + c_1)^{-\frac{2}{n}} - c_0(nDt + c_1)^{-\frac{3(1+\omega^{(m)})}{n}}}. \tag{33}$$

The above solutions satisfy the equation (29) identically, as expected.

The spatial volume (V) and expansion scalar (θ) of the model read as

$$V = a^3 = (nDt + c_1)^{\frac{3}{n}}, \tag{34}$$

$$\theta = 3H = 3D(nDt + c_1)^{-1}. \tag{35}$$

The anisotropy parameter $(\bar{A})$ and shear scalar (σ) of the model are given by

$$\bar{A} = \frac{1}{9H^2}\left[\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)^2 + \left(\frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right)^2\right] = \frac{2l^2}{3D^2}(nDt + c_1)^{-\frac{2(3-n)}{n}}, \tag{36}$$

$$\sigma^2 = \frac{3}{2}\bar{A}H^2 = l^2(nDt + c_1)^{\frac{-6}{n}}. \tag{37}$$

The value of DP (q) is found to be

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = n - 1, \tag{38}$$

which is a constant. The sign of q indicates whether the model inflates or not. A positive sign of q, i.e., $n > 1$ corresponds to the standard decelerating model whereas the negative sign of q, i.e., $0 < n < 1$ indicates inflation. The expansion of the Universe at a constant rate corresponds to $n = 1$, i.e., $q = 0$. Also, recent observations of SN Ia [1]−[5] reveal that the present Universe is accelerating and value of DP lies somewhere in the range $-1 < q < 0$. It follows that in the derived model, one can choose the values of DP consistent with the observations.

We observe that at $t = -c_1/nD$, the spatial volume vanishes while all other parameters diverge. Therefore, the model has a big bang singularity at $t = -c_1/nD$, which can be shifted to $t = 0$ by choosing $c_1 = 0$. The singularity is point type as the directional scale factors $A(t)$, $B(t)$ and $C(t)$ vanish at the initial moment.
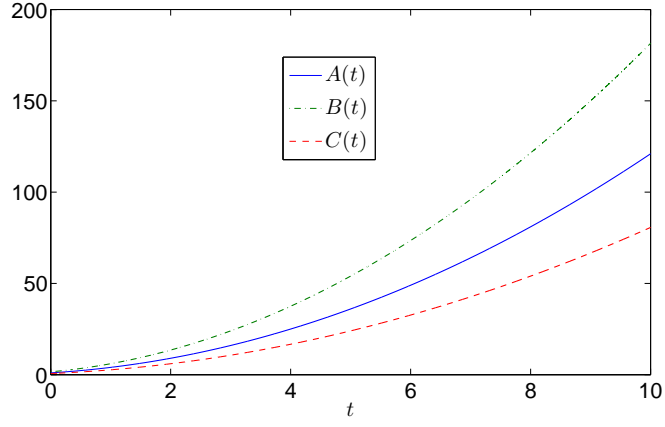
Figure 1: Scale factors vs. time with $m = 1.5$, $l = 0.3$, $D = 2$, $c_1 = 1$, $n = 0.5$.
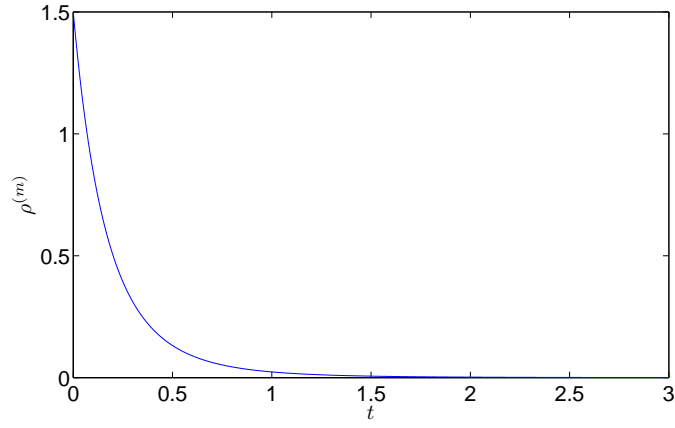

Figure 2: Matter energy density vs. time with $c_0 = 1.5$, $c_1 = 1$, $D = 2$, $n = 0.5$, $\omega^{(m)} = 0$.
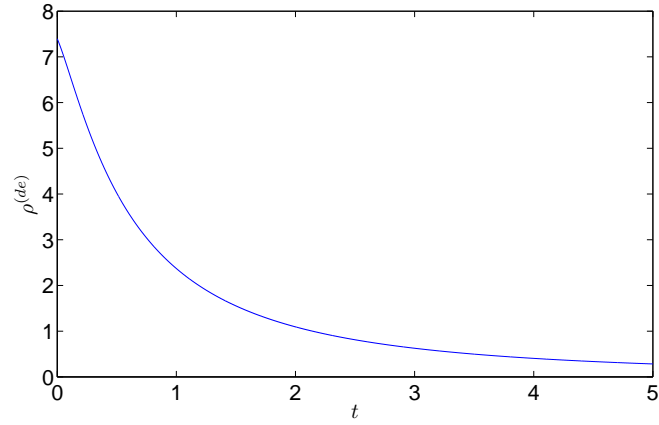

Figure 3: DE density vs. time with $c_0 = 1.5$, $c_1 = 1$, $D = 2$, $n = 0.5$, $\omega^{(m)} = 0$, $l = 1.2$, $\alpha = 0.7$.
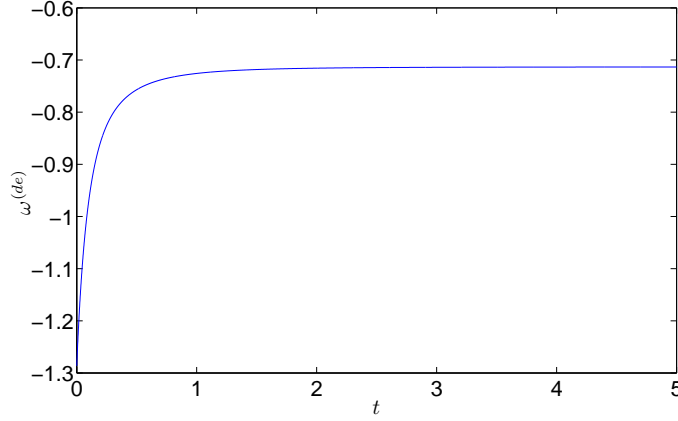
Figure 4: EoS parameter of DE vs. time with $c_0 = 1$, $c_1 = 0.05$, $D = 2$, $n = 0.5$, $\omega^{(m)} = 0$, $l = 1.2$, $\alpha = 0.7$.
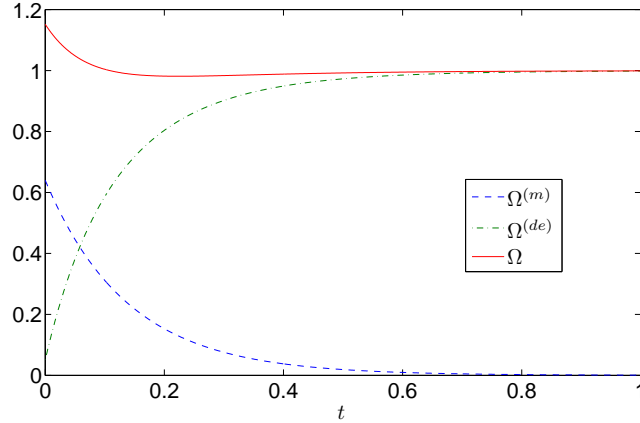

Figure 5: Density parameters vs. time with $D = 2.5$, $c_0 = 12$, $c_1 = 1$, $\omega^{(m)} = 0$, $n = 0.05$, $l = 1.2$, $\alpha = 0.7$.

Fig.1 shows that the directional scale factors monotonically increase with time. Thus, expansion of the Universe takes place in all the three spatial directions.

From **Fig. 2** and **Fig. 3**, we observe that $\rho^{(m)}$ as well as $\rho^{(de)}$ remain positive during the cosmic evolution. Therefore, the weak energy condition (WEC) as well as null energy condition (NEC) are obeyed in the derived model. Further, $\rho^{(m)}$ and $\rho^{(de)}$ decrease with time, and approach to small positive values at the present epoch.

The parameters H, θ and σ^2 start off with extremely large values, and continue to decrease with the expansion of the Universe. The anisotropy parameter $\bar{A}$ also decreases with the cosmic evolution provided $n < 3$. This shows that anisotropy of the model goes off during the cosmic evolution. The spatial volume grows with the cosmic time.

Fig. 4 depicts the variation of EoS parameter ($\omega^{(de)}$) versus cosmic time for accelerating phase of Universe ($q = -0.5$), as a representative case with appropriate choice of constants of integration and other physical parameters . The SN Ia data suggests that $-1.67 < \omega^{(de)} < -0.62$ [16] while the limit imposed on $\omega^{(de)}$ by a combination of SN Ia data (with CMB anisotropy) and galaxy clustering statistics is $-1.33 < \omega^{(de)} < -0.79$ [17]. **Fig. 4**, clearly shows that $\omega^{(de)}$ evolves within a range, which is in nice agreement with SN Ia and CMB observations.

The perfect fluid density parameter ($\Omega^{(m)}$) and DE density parameter ($\Omega^{(de)}$) are given by

$$\Omega^{(m)} = \frac{c_0}{3D^2}(nDt + c_1)^{\frac{2n - (1 + \omega^{(m)})}{n}}, \tag{39}$$

$$\Omega^{(de)} = 1 - \frac{l^2}{3D^2}(nDt + c_1)^{\frac{2n-6}{n}} - \frac{\alpha^2}{D^2}(nDt + c_1)^{\frac{2n-2}{n}} - \frac{c_0}{3D^2}(nDt + c_1)^{\frac{2n-(1+\omega^{(m)})}{n}}. \tag{40}$$

The overall density parameter (Ω) is given by

$$\Omega = \Omega^{(m)} + \Omega^{(de)} = 1 - \frac{l^2}{3D^2}(nDt + c_1)^{\frac{2n-6}{n}} - \frac{\alpha^2}{D^2}(nDt + c_1)^{\frac{2n-2}{n}}. \tag{41}$$

From equation (41), it is observed that for $0 < n < 1$, the overall density parameter (Ω) approaches to 1 at late times. Thus, the derived model predicts a flat Universe at the present epoch. **Fig. 5**, depicts the variation of density parameters versus cosmic time during the evolution of Universe. It is observed that initially Universe was matter dominated and later on DE dominates the evolution of Universe which is probably responsible for the accelerated expansion of present-day Universe.

3.2 DE Cosmology for $n = 0$

In this case, integration of (24) yields

$$a(t) = c_2 e^{Dt}, \tag{42}$$

where c_2 is a positive constant of integration.

The metric functions, therefore, read as

$$A(t) = c_2 e^{Dt}, \tag{43}$$

$$B(t) = mc_2 \exp\left(Dt - \frac{l}{3Dc_2^3}e^{-3Dt}\right), \tag{44}$$

$$C(t) = m^{-1}c_2 \exp\left(Dt + \frac{l}{3Dc_2^3}e^{-3Dt}\right). \tag{45}$$

Therefore, the model (1) becomes

$$ds^2 = -dt^2 + c_2^2 e^{2Dt}\left(dx^2 + m^2 e^{2\alpha x + hle^{-3Dt}}dy^2 + m^{-2}e^{2\alpha x - hle^{-3Dt}}dz^2\right). \tag{46}$$

where $h = \frac{-2}{3Dc_2^3}$.

The average Hubble's parameter, energy density of perfect fluid, DE density and EoS parameter of DE, for the model (46) are obtained as

$$H = D, \tag{47}$$

$$\rho^{(m)} = c_0 c_2^{-3(1+\omega^{(m)})} e^{-3D(1+\omega^{(m)})t}, \tag{48}$$

$$\rho^{(de)} = 3D^2 - l^2 c_2^{-6} e^{-6Dt} - 3\alpha^2 c_2^{-2} e^{-2Dt} - c_0 c_2^{-3(1+\omega^{(m)})} e^{-3D(1+\omega^{(m)})t}, \tag{49}$$

$$\omega^{(de)} = \frac{-3D^2 - l^2 c_2^{-6} e^{-6Dt} + \alpha^2 c_2^{-2} e^{-2Dt}}{3D^2 - l^2 c_2^{-6} e^{-6Dt} - 3\alpha^2 c_2^{-2} e^{-2Dt} - c_0 c_2^{-3(1+\omega^{(m)})} e^{-3D(1+\omega^{(m)})t}}. \tag{50}$$

The above solutions satisfy the equation (46) identically, as expected.

The spatial volume and expansion scalar of the model read as

$$V = c_2^3 e^{3Dt}, \tag{51}$$

$$\theta = 3D. \tag{52}$$

The DP is given by

$$q = -1. \tag{53}$$

The perfect fluid density parameter and DE density parameter are given by

$$\Omega^{(m)} = \frac{c_0 c_2^{-3(1+\omega^{(m)})}}{3D^2} e^{-3D(1+\omega^{(m)})t}, \tag{54}$$

$$\Omega^{(de)} = 1 - \frac{l^2 c_2^{-6}}{3D^2} e^{-6Dt} - \frac{\alpha^2 c_2^{-2}}{D^2} e^{-2Dt} - \frac{c_0 c_2^{-3(1+\omega^{(m)})}}{3D^2} e^{-3D(1+\omega^{(m)})t}. \tag{55}$$

The overall density parameter reads as

$$\Omega = 1 - \frac{l^2 c_2^{-6}}{3D^2} e^{-6Dt} - \frac{\alpha^2 c_2^{-2}}{D^2} e^{-2Dt}. \tag{56}$$

The anisotropic parameter and shear scalar of model (46) are given by

$$\bar{A} = \frac{2l^2 c_2^{-6}}{3D^2} e^{-6Dt}, \tag{57}$$

$$\sigma^2 = l^2 c_2^{-6} e^{-6Dt}. \tag{58}$$

Recent observations of SN Ia [1]−[5], suggest that the Universe is accelerating in its present state of evolution. It is believed that the way Universe is accelerating presently; it will expand at the fastest possible rate in future and forever. For $n = 0$, we get $q = -1$; incidentally this value of DP leads to $dH/dt = 0$, which implies the greatest value of Hubble's parameter and the fastest rate of expansion of the Universe. Therefore, the derived model can be utilized to describe the dynamics of the late time evolution of the actual Universe. So, in what follows, we emphasize upon the late time behavior of the derived model.

From Equation (50), we observe that $\omega^{(de)} \approx -1$ for sufficiently large time t. Therefore, the late time dynamics of EoS parameter $\omega^{(de)}$ represents the vacuum fluid dominated Universe, which is mathematically equivalent to cosmological constant. Further, at late times, we have

$\rho^{(m)} \approx 0,$

$\rho^{(de)} \approx 3D^2,$

$\bar{A} \approx 0.$

Thus, the ordinary matter density becomes negligible whereas the accelerated expansion of Universe continues with non-zero and constant DE density, which is consistent with recent observations. Also the late time dynamics of the derived model shows that the anisotropy of the Universe damps out and Universe becomes isotropic. From equation (56), it is observed that for sufficiently large time, the overall density parameter approaches to 1, i.e., $\Omega \approx 1$. Thus, the model predicts a flat Universe.

4 Concluding Remarks

In this paper, we have investigated the role of DE with variable EoS parameter in the evolution of Universe within the framework of a spatially homogeneous and anisotropic Bianchi-V space-time by taking into account the special law of variation of average Hubble parameter, which yields a constant value of DP. The main features of the work are as follows:

- The models are based on exact solutions of the field equations.

- In the present models, the matter energy density and DE density remain positive. Therefore, the weak and null energy conditions are satisfied, which in turn imply that the derived models are physically realistic.

- The singular model ($n \neq 0$) seems to describe the dynamics of Universe from big bang to the present epoch while the non-singular model ($n = 0$) seems reasonable to project dynamics of future Universe.

- The age of Universe, in the singular model, is given by

$$T_0 = \left(\frac{1}{D}\right) H_0^{-1}$$

which differs from the present estimate, i.e., $T_0 = H_0^{-1} \approx 14 Gyr$. But if we take $D = 1$, the model ($n \neq 0$) is in good agreement with the present age of Universe.

- The EoS parameter of DE evolves within the range predicted by the observations.

- The DE dominates the Universe at the present epoch, which may be attributed to the current accelerated expansion of the Universe.

- The Universe acquires flatness with the dominance of DE (see, **Fig.5**). Thus, flatness of the Universe seems a natural consequence of DE.

Hypothetical DE is the most popular way of explaining why the Universe is expanding at an ever increasing rate. DE plays a massive part in shaping our reality however nobody seems certain of what the dang stuff actually is. Future space missions hope to solve this mystery and shake up our current understanding of the Universe.

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